On exact Pleijel's constant for some domains

Vladimir Bobkov

NTIS – New Technologies for the Information Society
Faculty of Applied Sciences, University of West Bohemia
Univerzitní 8, 306 14 Plzeň, Czech Republic
e-mail: bobkov@kma.zcu.cz

Abstract

Let λ_k and φ_k be the k-th eigenvalue and an associated eigenfunction, respectively, of the Dirichlet Laplacian on a bounded domain $\Omega \subset \mathbb{R}^2$. Denote by $\mu(\varphi_k)$ the number of nodal domains of φ_k. Courant's nodal domain theorem asserts that $\mu(\varphi_k) \leq k$ for any k. Pleijel obtained the following refinement of this fact:

$$Pl(\Omega) := \limsup_{k\to\infty} \frac{\mu(\varphi_k)}{k} \leq \frac{4}{j_{0,1}^2} = 0.69166\ldots$$

Here, $Pl(\Omega)$ is called Pleijel constant of Ω.

In the present talk, we discuss explicit expressions and values of the Pleijel constant $Pl(\Omega)$ for several domains Ω with separable geometries, such as a disk, annuli, and their sectors. Consideration of the case of annuli required the development of the theory of zeros of cross-products of Bessel functions, and revealed natural but open problems on multiplicity of corresponding eigenvalues. The talk is based on the preprints arXiv:1802.04357 and arXiv:1803.09972.