

Multiple critical points of saddle geometry functionals

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Abstract

We study the multiplicity of critical points for continuously differentiable functionals on real Banach spaces. We prove that a functional which satisfies the assumptions of the Saddle Point Theorem and moreover is bounded from below has at least three critical points. Apparently, there is a global minimizer and a saddle point and we show the existence of a third critical point. The idea of the proof is based on the minus-gradient flow. This result is closely related to the three critical points theorem of H. Brezis and L. Nirenberg which assumes a local linking. Finally, we apply the result on the Dirichlet problem for semilinear stationary PDEs. The analysis includes, for example, the existence of multiple stationary solutions of bistable (or Allen-Cahn) equation and semipositone problems.