Regularity on a fixed set

Radek Cibulka

Department of Mathematics, Faculty of Applied Sciences
University of West Bohemia in Pilsen
Czech Republic
e-mail: cibi@kma.zcu.cz

Abstract

Ioffe's criterion and various reformulations of it have become a standard tool in proving theorems guaranteeing metric regularity of a (set-valued) mapping. First, we demonstrate that one should always use directly the so-called general criterion which follows, for example, from Ekeland's variational principle, and that there is no need to make a detour through the slope-based consequences of this general statement. Second, we argue that when proving perturbation stability results, in the spirit of Lyusternik-Graves theorem, there is no need to employ the concept of a lower semi-continuous envelope even in the case of an incomplete target space. The gist is to use the "correct" function to which Ekeland's variational principle is applied; namely, the distance function to the graph of the set-valued mapping under consideration. This approach originates in the notion of graphical regularity introduced by L. Thibault, which is equivalent to the property of metric regularity. Our criteria cover also both metric subregularity and metric semiregularity, which are weaker properties obtained by fixing one of the points in the definition of metric regularity.