The space of Hardy-weights for quasilinear equations: Maz'ya-type characterization and sufficient conditions for the existence of minimizers

Ujjal Das

Technion-Israel Institute of Technology
Haifa, Israel
e-mail: ujjal.rupam.das@gmail.com

Abstract

Let $p \in (1, \infty)$ and $\Omega \subset \mathbb{R}^N$ be a domain. Let $A := (a_{ij}) \in L^\infty_{\mathrm{loc}}(\Omega; \mathbb{R}^{N\times N})$ be a symmetric and locally uniformly positive definite matrix. Set $|\xi|_A^2 := \sum_{i,j=1}^{N} a_{ij}(x)\xi_i\xi_j$, $\xi \in \mathbb{R}^N$, and let V be a given potential in a certain local Morrey space. We assume that the energy functional

$$Q_{p,A,V}(\phi) := \int_\Omega [|\nabla\phi|_A^p + V|\phi|^p]\mathrm{dx}$$

is nonnegative in $W^{1,p}(\Omega) \cap C_c(\Omega)$.

We introduce a generalized notion of $Q_{p,A,V}$-capacity and characterize the space of all Hardy-weights for the functional $Q_{p,A,V}$, extending Maz'ya's well-known characterization of the space of Hardy-weights for the p-Laplacian. In addition, we provide various sufficient conditions on the potential V and the Hardy-weight g such that the best constant of the corresponding variational problem is attained in an appropriate Beppo-Levi space.