On a critical Kirchhoff type problem in high dimension

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Abstract

Nonlocal boundary value problems of the type

$$\begin{cases} -\left(a + b\int_\Omega |\nabla u|^2 dx\right)\Delta u = f(x,u), & \text{in } \Omega \\ u = 0, & \text{on } \partial\Omega \end{cases}$$

are related to the stationary version of the Kirchhoff equation

$$\frac{\partial^2 u}{\partial t^2} - \left(a + b\int_\Omega |\nabla u|^2 dx\right)\Delta u = f(t,x,u),$$

first proposed by Kirchhoff in 1883 to describe the transversal oscillations of a stretched string.

We consider the case when Ω is a bounded domain of $\mathbb{R}^N$ with $N > 4$ and the nonlinearity f is of the type $f(x,u) = |u|^{2^*-2}u + \lambda g(x,u)$, being 2^* the Sobolev critical exponent, g a subcritical perturbation, λ a positive parameter.

In this talk, we will show that the interaction between the Kirchhoff operator and the critical term leads to some variational properties of the energy functional as the sequential weak lower semicontinuity and the Palais–Smale condition provided a and b satisfy a suitable constraint.

Then, through a careful analysis of the fiber maps associated to the energy functional, we will deduce the existence, non existence and multiplicity of solutions of our problem when the parameters a, b, λ vary in appropriate intervals. When the nonlinearity g is a pure power term, i.e. $g(x,u) = |u|^{p-2}u$ for some $p \in (2, 2^*)$, through a detailed study of the Nehari sets associated to the problem, we will show the existence of two critical hyperbolas on the plane (a,b) that separates the plane into regions where the energy functional exhibits distinct topological properties.

Based on the papers [1–3].

References

[1] F. FARACI, Cs. FARKAS, *On an open question of Ricceri concerning a Kirchhoff-type problem*, Minimax Theory and its Applications, 4 (2019).
[2] F. FARACI, Cs. FARKAS, *On a critical Kirchhoff-type problem*, Nonlinear Anal., 192 (2020).
[3] F. FARACI, K. SILVA, *On the Brezis-Nirenberg problem for a Kirchhoff type equation in high dimension*, Calc. Var. Partial Differential Equations, 60 (2021).