New mathematical model of the fluid flow in the porous medium layered over inclined impermeable bed

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Abstract

We propose a new mathematical model of groundwater flow in porous medium layered over inclined impermeable bed. Novelty of our approach consists in considering nonlinear constitutive law of the power type. In its full generality, this is a free-surface problem. To obtain analytically tractable model, we use a generalized Dupuit-Forchheimer assumption for inclined impermeable bed, as it is customary in this situation. Since we use nonlinear constitutive law of the power type instead of Darcy law, we arrive at parabolic partial differential equation which is a generalization of the classical Boussinesq equation. Thus a p-Laplacian-like differential operator is introduced into the Boussinesq equation. Unlike in the classical case of the Boussinesq equation, the convective term cannot be set aside from the main part of the diffusive term and remains incorporated within it.

In the sequel of the talk, we will analyze qualitative properties of the stationary solutions of our model. In particular, we will study regularity of weak solutions and the validity of strong maximum principle for the following boundary value problem

$$
\begin{aligned}
&-\frac{\mathrm{d}}{\mathrm{d}x}\left[(u(x)+H)\left|\frac{\mathrm{d}u}{\mathrm{d}x}(x)\cos(\varphi)+\sin(\varphi)\right|^{p-2}\left(\frac{\mathrm{d}u}{\mathrm{d}x}(x)\cos(\varphi)+\sin(\varphi)\right)\right]\\
&=f(x)\,, \qquad\qquad x\in(-1,1)\,,\\
&u(-1)=u(1)=0\,,
\end{aligned}
$$

where $p>2$, $H>0$, $\varphi\in(0,\pi/2)$, $f\geq 0$, $f\in L^\infty(-1,1)$. As a starting point, we use regularity theory for quasilinear elliptic problems developped by Lieberman, which we further refine for the ODE case. Then we use methods based on the linearization of the p-Laplacian-type problems in the vicinity of known solution, error estimates, and analysis of the Green function of the linearized problem. This is joint work with Petr Girg.