Fast MATLAB evaluation of nonlinear energies using FEM in 2D and 3D: Nodal elements

Alexej Moskovka

Department of Mathematics, Faculty of Applied Sciences
University of West Bohemia in Pilsen
Czech Republic
e-mail: alexmos@kma.zcu.cz

Abstract

Nonlinear energy functionals appearing in the calculus of variations can be discretized by the finite element (FE) method and formulated as a sum of energy contributions from local elements. A fast evaluation of energy functionals containing the first order gradient terms is a central part of this contribution. We describe a vectorized implementation using the simplest linear nodal (P1) elements in which all energy contributions are evaluated all at once without the loop over triangular or tetrahedral elements. Furthermore, in connection to the first-order optimization methods, the discrete gradient of energy functional is assembled in a way that the gradient components are evaluated over all degrees of freedom all at once. The key ingredient is the vectorization of exact or approximate energy gradients over nodal patches. It leads to a time-efficient implementation at higher memory-cost. Provided codes in MATLAB related to 2D/3D hyperelasticity and 2D p-Laplacian problem are available for download and structured in a way it can be easily extended to other types of vector or scalar forms of energies.